

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**Math 10550, Practice Final Exam, December**

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 2 hours.
- Be sure that your name is on this page.
- Be sure that you have all 25 problems.
- This is the only page you need to hand in.

Please mark your answers with an **X**!      Do NOT circle them!

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25.  a  b  c  d  e

12.  a  b  c  d  e

Final Exam: \_\_\_\_\_

13.  a  b  c  d  e

Previous Total: \_\_\_\_\_

14.  a  b  c  d  e

Course Total: \_\_\_\_\_

**1.(6 pts.)** Compute  $\lim_{x \rightarrow 2^-} \frac{x^2 - 4}{x^2 - 5x + 6}$ .

- (a) -4      (b)  $\infty$       (c) 0      (d) 1      (e)  $-\infty$

**2.(6 pts.)** Compute  $\lim_{x \rightarrow 0^+} \frac{x^2 - 9}{\sin x}$ .

- (a) Does not exist and is not  $\infty$  or  $-\infty$ .      (b)  $\infty$   
(c) 0      (d) -9  
(e)  $-\infty$

**3.(6 pts.)** Evaluate  $\lim_{x \rightarrow \infty} (\sqrt{x^2 - x} - \sqrt{x^2 + 5x})$ .

- (a) 0                          (b) 3                          (c) -6  
(d) Does not exist            (e) -3

**4.(6 pts.)** For what constant  $a$  is the function  $f$  given by

$$f(x) = \begin{cases} ax + 1 & x < 0 \\ x^2 + 1 & x \geq 0 \end{cases}$$

differentiable everywhere?

- (a)  $a = 2$                           (b)  $a = 0$   
(c)  $a = 1$                               (d) Any value of  $a$   
(e) No value of  $a$

**5.(6 pts.)** Compute  $\lim_{x \rightarrow 0} \frac{\tan 2x}{\sin 3x}$ .

- (a) 1/3      (b) 2      (c) 0      (d) 2/3      (e) 1

**6.(6 pts.)** Compute  $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + x + 1}}{3x - 1}$ .

- (a) 0      (b) -2/3      (c) 2/3      (d) 1/3      (e) -1/3

**7.(6 pts.)** Compute the tangent line to the ellipse given by the equation  $x^2 + 4y^2 = 5$  at the point  $(1, -1)$

- (a)  $y = \frac{1}{2}x - \frac{3}{2}$       (b) The tangent line does not exist.
- (c)  $y = \frac{1}{4}x - \frac{5}{4}$       (d)  $y = \frac{1}{4}x - \frac{3}{4}$
- (e)  $y = -\frac{1}{4}x - \frac{3}{4}$

**8.(6 pts.)** Let  $F(x) = f(g(x))$ . Compute  $F'(2)$  using the following information:  
 $f(-1) = -3, f(2) = 12, g(-1) = -7, g(2) = -1,$   
 $f'(-1) = 2, f'(2) = 8, g'(-1) = -1, g'(2) = 5.$

- (a) 10      (b) -15      (c) 40      (d) 2      (e) 52

**9.(6 pts.)** For  $y = (\sin 4x)^8$ , compute  $y'$ .

- (a)  $32(\cos 4x)^7$       (b)  $8(\cos 4x)^7$   
(c)  $8(\sin 4x)^7$       (d)  $32(\sin 4x)^7$   
(e)  $32(\sin 4x)^7 \cos 4x$

**10.(6 pts.)** How many inflection points does the curve  $y = \frac{x^5}{5} + \frac{x^4}{4}$  have?

- (a) 1      (b) 0      (c) 3      (d) 2      (e) 4

**11.(6 pts.)** Compute the derivative  $y'$  for the curve  $\sqrt{x^2 + y^2} = 2 + y$  at the point  $x = 4$ ,  $y = 3$ .

- (a)  $2/11$       (b)  $-2$       (c)  $2$       (d)  $0$       (e)  $-2/11$

**12.(6 pts.)** A kite 100 ft above the ground is flying horizontally (away from its holder) with a speed of 16ft/sec. At what rate is the angle between the string and the horizontal direction changing, when 200 ft of the string have been let out?

- (a)  $\frac{\pi}{50}$  radian/second      (b)  $\frac{1}{25}$  radian/second  
(c)  $\frac{1}{50}$  radian/second      (d)  $-\frac{1}{25}$  radian/second  
(e)  $-\frac{1}{50}$  radian/second

**13.(6 pts.)** Find the linearization of  $f(x) = \sqrt{10 - x^2}$  at  $a = -1$ .

(a)  $L(x) = \frac{2}{3}(x + 1) + 3$

(b)  $L(x) = -\frac{2}{3}(x + 1) + 3$

(c)  $L(x) = x + 4$

(d)  $L(x) = -\frac{1}{3}(x + 1) + 3$

(e)  $L(x) = \frac{1}{3}(x + 1) + 3$

**14.(6 pts.)** Find all local maxima and minima of the function  $f(x) = 2|x| - x^2 - 1$ .

- (a) Local maxima:  $(x, y) = (-1, 0)$  and  $(x, y) = (1, 0)$ , local minimum  $(x, y) = (0, -1)$ .
- (b) Only local minimum at  $(x, y) = (0, -1)$ , no local maxima.
- (c) Local maximum:  $(x, y) = (-1, 0)$ , local minimum  $(x, y) = (0, -1)$ .
- (d) No local maxima or minima, because the function  $|x|$  has no derivative at  $x = 0$ .
- (e) Local maxima:  $(x, y) = (-1, 0)$  and  $(x, y) = (1, 0)$ , no local minimum.

**15.(6 pts.)** Find all asymptotes of the curve  $y = \frac{2x^2 + x + 1}{x - 1}$ .

- (a) vertical asymptote  $x = 1$ , no other asymptotes.
- (b) slant asymptote  $y = 2x + 1$ , vertical asymptote  $x = 1$ , no horizontal asymptotes.
- (c) horizontal asymptotes  $y = 2$ , slant asymptote  $y = 2x + 3$ , no vertical asymptotes.
- (d) slant asymptote  $y = 2x + 3$ , vertical asymptote  $x = 1$ , no horizontal asymptotes.
- (e) horizontal asymptotes  $y = 2$ , vertical asymptote  $x = 1$ , no slant asymptotes.

**16.(6 pts.)** Find **all** the points on the hyperbola  $y^2 - x^2 = 4$  that are closest to the point  $(2, 0)$ .

- (a)  $(1, \pm 5)$
- (b)  $(1, \pm \sqrt{5})$
- (c)  $(-1, \sqrt{5})$
- (d)  $(1, \sqrt{5})$
- (e)  $(\sqrt{5}, 1)$

**17.(6 pts.)** A page of a book is to have a total area of 150 square inches, with 1 inch margins at the top and sides, and a 2 inch margin at the bottom. Find the dimensions in inches of the page which will have the largest print area.

- (a)  $3\sqrt{7} \times \frac{50}{\sqrt{7}}$       (b)  $5 \times 30$       (c)  $11\frac{7}{13} \times 13$   
(d)  $5\sqrt{3} \times \frac{30}{\sqrt{3}}$       (e)  $10 \times 15$

**18.(6 pts.)** Newton's method is to be used to find a root of the equation

$$x^3 - x - 1 = 0.$$

If  $x_1 = 1$ , find  $x_2$ .

- (a) 1.50      (b) 0.95      (c) 3      (d) 1.35      (e) 1.75

**19.(6 pts.)** Express the limit below as a definite integral.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{4n} \sec^2 \left( \frac{i\pi}{4n} \right)$$

(a)  $\int_0^1 \sec^2 \left( \frac{\pi}{4}x \right) dx$

(b)  $\frac{\pi}{4} \int_0^{\pi/4} \sec^2(x) dx$

(c)  $\int_0^{\pi/4} \sec^2 \left( \frac{\pi}{4} \right) dx$

(d)  $\int_0^{\pi/2} \sec^2(x) dx$

(e)  $\int_0^{\pi/4} \sec^2(x) dx$

**20.(6 pts.)** If  $f(x) = \int_0^{5x} \cos(u^2) du$ , find  $f'(x)$ .

(a)  $-\cos(5x^2)$

(b)  $5 \cos(25x^2)$

(c)  $-25 \cos(5x^2)$

(d)  $5 \cos(5x^2)$

(e)  $-5 \cos(25x^2)$

**21.(6 pts.)** Evaluate the integral  $\int_0^{\sqrt{\pi}} x \sin(x^2) dx$ .

- (a)  $\frac{\pi}{4}$       (b) 2      (c)  $\frac{1}{4}$       (d)  $1 - \frac{1}{\pi}$       (e) 1

**22.(6 pts.)** Which of the following integrals give the area of the region below the curve  $y = 2x$  and above the curve  $y = x^2 - 4x$ ?

(a)  $\int_0^4 ((x^2 - 4x) - 2x) dx$

(b)  $\int_0^6 ((x^2 - 4x) - 2x) dx$

(c)  $\int_0^6 (2x - (x^2 - 4x)) dx$

(d)  $\int_0^4 (2x - (x^2 - 4x)) dx$

(e)  $\int_0^4 (2x - (x^2 - 4x)) dx + \int_4^6 ((x^2 - 4x) - 2x) dx$

**23.(6 pts.)** An area in  $xy$  plane bounded by the curves  $y = 0$  and  $y = x - x^2$ . If we rotate this area about  $x = 7$ , which integral below gives the volume?

(a)  $\pi \int_0^{1/4} (x - x^2)^2 dx$

(b)  $2\pi \int_0^1 (7 - x)(x - x^2) dx$

(c)  $2\pi \int_0^\pi (x - x^2 - 7) dx$

(d)  $\pi \int_0^1 (x - x^2)^2 dx$

(e)  $2\pi \int_0^1 (x - 7)(x - x^2) dx$

**24.(6 pts.)** The plane region bounded by the curves  $y = 2$  and  $y = 2 + 2x - x^2$  is rotated about the  $x$  axis. Which integral below gives the volume?

(a)  $\pi \int_0^2 \left(4 - (2 + 2x - x^2)^2\right) dx$

(b)  $\pi \int_0^2 \left((2 + 2x - x^2)^2 - 4\right) dx$

(c)  $2\pi \int_0^2 \left((2 + 2x - x^2) - 2\right) dx$

(d)  $\pi \int_0^1 \left((2 + 2x - x^2)^2 - 4\right) dx$

(e)  $\pi \int_0^1 \left(4 - (2 + 2x - x^2)^2\right) dx$

**25.(6 pts.)** The function  $f(x) = \sqrt{16 - 2x}$  is continuous on the interval  $[0, 8]$ . Which number below is its average value on this interval?

- (a)  $\frac{8}{3}$       (b)  $\frac{64}{3}$       (c)  $\frac{8}{3}\sqrt{8}$   
(d)  $\frac{16}{3}$       (e)  $-\frac{8}{3}$

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